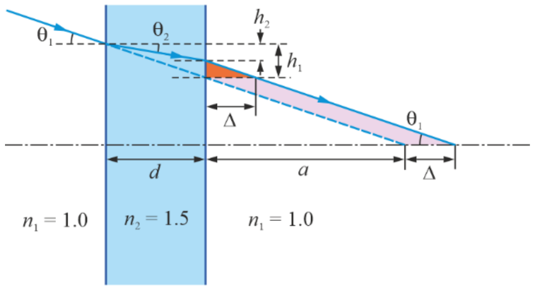
# Answers to Homework 1

## Problem 1.1




Consider the general geometry shown in the above figure. A light ray entering and exiting a slab, of refractive index $n_2$ that is different from that of the ambient $n_1$ and a thickness $d$, will intersect with the reference axis at a shifted location by a distance $\Delta$ from its original intersection point of distance $a$ from the exit surface. From simple geometry of the light purple parallelogram, this shift $\Delta$ is equal to the base of the orange triangle, whose height is

$$h = h_1 - h_2,$$

where

$$h_1 = d\tan\theta_1,$$

$$h_2 = d\tan\theta_2,$$

and based on the Snell's Law,

$$n_1\sin\theta_1 = n_2\sin\theta_2,$$

which leads to

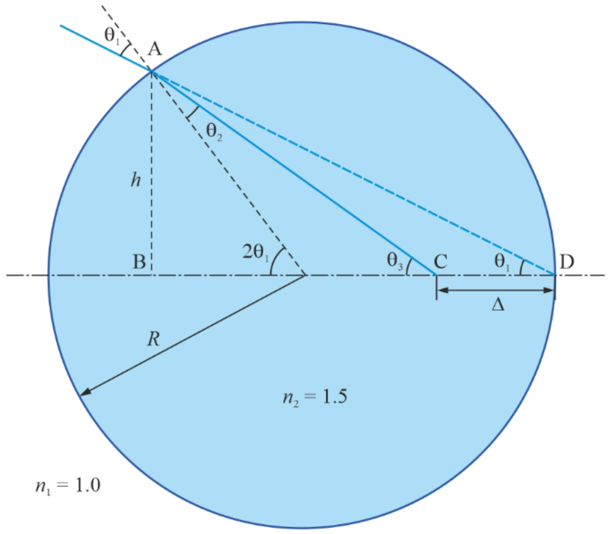
$$\theta_2 = \arcsin\left(\frac{n_1}{n_2}\sin\theta_1\right).$$

Therefore, we can evaluate the shift as a function of $\theta_1$, and $d$:

$$\Delta(\theta_1, d) = h/\tan\theta_1 = d\,\frac{\tan\theta_1 - \tan\theta_2}{\tan\theta_1} = d\left\{1 - \tan\left[\arcsin\left(\frac{n_1}{n_2}\sin\theta_1\right)\right]\Big/\tan\theta_1\right\}$$

As a comparison, the difference between the two interaction points at $\theta_1 = 30°$ and $\theta_1 = 1°$ when $d = 1\text{ cm}$ is then $\Delta(30°, 1\text{ cm}) - \Delta(1°, 1\text{ cm}) = 0.0542\text{ cm}$.

**Problem 1.2**




Consider the general geometry shown in the above figure. A light ray incident upon a sphere, of refractive index $n_2$ that is different from that of the ambient $n_1$ and a radius $R$, will intersect with the reference axis passing the center of the sphere at a shifted location by a distance $\Delta$ from its original intersection point of distance $R$ from the center. From simple geometry, we have

$$h = R\sin 2\theta_1$$

and

$$\theta_3 = 2\theta_1 - \theta_2,$$

where the Snell's Law requires that

$$\theta_2 = \arcsin\left(\frac{n_1}{n_2}\sin\theta_1\right).$$

The distance $\Delta$ is the difference between the base of triangle ABD and the base of triangle ABC. Therefore, we can evaluate the shift as a function of $\theta_1$, and $R$:

$$\Delta(\theta_1, R) = \frac{h}{\tan\theta_1} - \frac{h}{\tan\theta_3} = R\sin 2\theta_1\left[\cot\theta_1 - \cot\left(2\theta_1 - \arcsin\frac{n_1\sin\theta_1}{n_2}\right)\right].$$

As a comparison, the difference between the two interaction points at $\theta_1 = 30°$ and $\theta_1 = 1°$ when $R = 0.5\ \text{cm}$ is then $\Delta(30°, 0.5\ \text{cm}) - \Delta(1°, 0.5\ \text{cm}) = -0.0899\ \text{cm}$.

**NOTE:** $\Delta$ does not necessarily cause aberrations. If $\Delta$ is consistent for rays of all angles, the system is still considered aberration-free. It is the inconsistency in $\Delta$ for rays of different angles that causes aberrations.